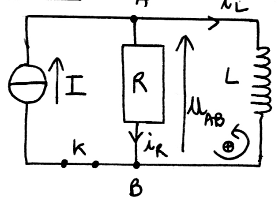
EXE3-1



- Loi des noeuds en A : $I = i_R + i_L$ (1)
- Maille $(A,\{R\},B,\{L\},A)$: $-R i_R + L\dfrac{di_L}{dt} = 0$

d'où $i_R = \dfrac{L}{R}\dfrac{di_L}{dt}$ (2)

- (1) $\xrightarrow{(2)}$ $I = \dfrac{L}{R}\dfrac{di_L}{dt} + i_L$

d'où i_L vérifie une éq diff du 1er ordre à coeff constants avec 2d membre : $\dfrac{di_L}{dt} + \dfrac{R}{L} i_L = I\dfrac{R}{L}$

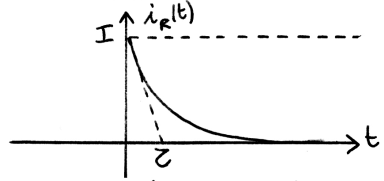
Posons $\boxed{\tau = \dfrac{L}{R}}$ alors : $\dfrac{di_L}{dt} + \dfrac{i_L}{\tau} = \dfrac{I}{\tau}$ solut° $i_L(t) = i_{L_0} + i_{L_P}$

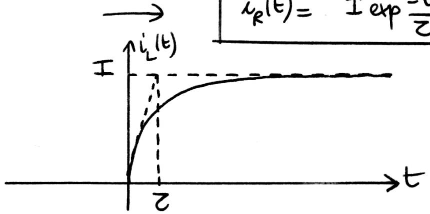
avec $i_{L_P} = c^{te} = I$ et $i_{L_0}(t) = A\exp\dfrac{-t}{\tau}$ D'où $i_L(t) = A\exp\dfrac{-t}{\tau} + I$

à $t=0$ on a $0 = i_L(0^-) = i_L(0^+) = A + I$ d'où $A = -I$ et

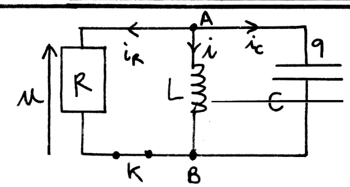
$$\boxed{i_L(t) = I\left(1 - \exp\frac{-t}{\tau}\right) \qquad i_R(t) = I\exp\frac{-t}{\tau}}$$

$i_R = \tau\dfrac{di_L}{dt}$





EXE3-2 Circuit RLC Parallèle



1) $\begin{cases} i_C = \dfrac{dq}{dt}, \quad i_R = \dfrac{u}{R} \quad \text{et} \quad i_R + i_C + i = 0 \quad (1) \\ u = R i_R = L\dfrac{di}{dt} = \dfrac{q}{C} \quad (2) \end{cases}$

(1) → $\dfrac{u}{R} + \dfrac{dq}{dt} + i = 0$

Or (2) ⇒ $\begin{cases} q = LC\dfrac{di}{dt} \\ u = L\dfrac{di}{dt} \end{cases}$ d'où $\dfrac{L}{R}\dfrac{di}{dt} + LC\dfrac{d^2i}{dt^2} + i = 0$ (1) → $\dfrac{d^2i}{dt^2} + \dfrac{1}{RC}\dfrac{di}{dt} + \dfrac{1}{LC}i = 0$

en introduisant $Q_0 = RC\omega_0$ et $\omega_0^2 = \dfrac{1}{LC}$

on arrive à $\boxed{\dfrac{d^2i}{dt^2} + \dfrac{\omega_0}{Q_0}\dfrac{di}{dt} + \omega_0^2 i = 0}$ (Rque $Q_0 = RC\omega_0 = \dfrac{R}{L\omega_0}$)

Rque l'expression est l'INVERSE du facteur de Qualité d'un RLC SÉRIE.

2) $\dfrac{d^2i}{dt^2} + \dfrac{\omega_0}{Q_0}\dfrac{di}{dt} + \omega_0^2 i = 0 \xrightarrow{2\lambda = \frac{1}{Q_0}} \boxed{\dfrac{d^2i}{dt^2} + 2\lambda\omega_0\dfrac{di}{dt} + \omega_0^2 i = 0}$ (E)

$\Delta' = b'^2 - ac = \lambda^2\omega_0^2 - \omega_0^2 = \omega_0^2(\lambda^2 - 1)$

3 Cas : a) Rég. Apériodique : $\Delta' > 0$ $\lambda > 1$

→ 2 racines réelles négatives

$r_1 = \dfrac{-b' - \sqrt{\Delta'}}{a} = -\lambda\omega_0 - \omega_0\sqrt{\lambda^2 - 1} < 0$

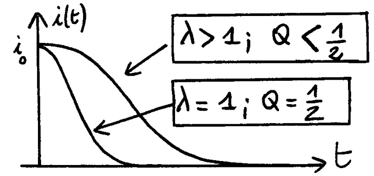
$r_2 = \dfrac{-b' + \sqrt{\Delta'}}{a} = -\lambda\omega_0 + \omega_0\sqrt{\lambda^2 - 1} < 0$ → $\boxed{i(t) = A_1 e^{r_1 t} + A_2 e^{r_2 t}}$

$i_0 = i(0^-) = i(0^+)$ Continuité de l'intensité traversant une bobine → $i(0) = i_0 = A_1 + A_2$

On a $u(t) = L\dfrac{di}{dt} = L(A_1 r_1 e^{r_1 t} + A_2 r_2 e^{r_2 t})$ avec $u(0^-) = u(0^+) = 0 = L(A_1 r_1 + A_2 r_2)$

continuité de la tension aux bornes de C

d'où $\begin{cases} A_1 + A_2 = i_0 & (1) \\ A_1 r_1 + A_2 r_2 = 0 & (2) \end{cases}$ (1)r_1 − (2) ⇒ $\boxed{A_2 = \dfrac{i_0 r_1}{r_1 - r_2}}$ $\boxed{A_1 = \dfrac{-i_0 r_2}{r_1 - r_2}}$



b) Régime Critique : $\Delta' = 0$ $\lambda = 1$ $Q = \frac{1}{2}$ 1 seule racine (<0) $r = \dfrac{-b'}{a} = -\lambda\omega_0$

$i(t) = (A_1 + A_2 t)e^{-\lambda\omega_0 t}$ $i(0^-) = i_0 = i(0^+) = A_1$ → $A_1 = i_0$

$u(t) = L(-A_1\lambda\omega_0 + A_2 - A_2\lambda\omega_0 t)e^{-\lambda\omega_0 t}$ $u(0^-) = 0 = u(0^+) = L(-A_1\lambda\omega_0 + A_2)$ → $A_2 = A_1\lambda\omega_0$

$$\boxed{i(t) = i_0(1 + \lambda\omega_0 t)e^{-\lambda\omega_0 t}}$$

c) Rég. Pseudo Périodique : $\Delta' < 0$ $\lambda < 1$ $Q > \frac{1}{2}$ 2 racines cplexes conjuguées :

$\Delta' = \omega_0^2(\lambda^2 - 1) = j^2\omega_0^2(1 - \lambda^2)$ → $r_1 = -\lambda\omega_0 - j\omega_0\sqrt{1 - \lambda^2}$ $r_2 = -\lambda\omega_0 + j\omega_0\sqrt{1 - \lambda^2}$

$\boxed{\tau \equiv \dfrac{1}{\lambda\omega_0} \qquad \omega \equiv \omega_0\sqrt{1 - \lambda^2}}$ → solut° $i(t) = (A_1\cos\omega t + A_2\sin\omega t)e^{-\frac{t}{\tau}}$

$u(t) = \left[\left(\dfrac{-A_1}{\tau} + A_2\omega\right)\cos\omega t + \left(-\dfrac{A_2}{\tau} - A_1\omega\right)\sin\omega t\right]e^{-\frac{t}{\tau}}$

$i_0 = i(0^-) = i(0^+) = A_1$ → $A_1 = i_0$

$0 = u(0^-) = u(0^+) = -\dfrac{A_1}{\tau} + A_2\omega$ → $A_2 = \dfrac{A_1}{\omega\tau}$

$$\boxed{i(t) = i_0\left(\cos\omega t + \frac{\sin\omega t}{\tau\omega}\right)\exp\left(-\frac{t}{\tau}\right)}$$