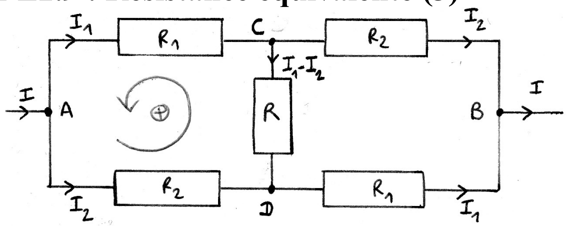
Ex-E2.9 : Résistance équivalente (3)



1) a) Lois de Kirchoff :

Loi des noeuds en A (ou B) $\boxed{I = I_1 + I_2 \quad (1)}$

Par symétrie $\boxed{\begin{array}{l} I_{A\to D} = I_2 = I_{C\to B} \\ I_{A\to C} = I_1 = I_{D\to B} \end{array}}$

Loi des noeuds en D (ou C) : $\boxed{I_{C\to D} = I_1 - I_2}$

Loi des mailles pour (A,D,C) : $-R_2 I_2 + R(I_1 - I_2) + R_1 I_1 = 0$

$\hookrightarrow \boxed{(R_1+R)I_1 - (R+R_2)I_2 = 0 \quad (2)}$

Système $\left\{\begin{array}{l} (1)\quad I_1 + I_2 = I \\ (2)\quad (R_1+R)I_1 - (R+R_2)I_2 = 0 \end{array}\right.$

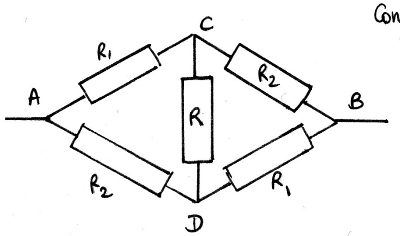
→ par substitution :

$\left\{\begin{array}{l} (1)\quad I_1 = I - I_2 \\ (2)\quad (R_1+R)(I - I_2) - (R+R_2)I_2 = 0 \end{array}\right. \rightarrow \boxed{\begin{array}{l} I_2 = \dfrac{R_1+R}{2R+R_1+R_2}\, I \\ I_1 = \dfrac{R_2+R}{2R+R_1+R_2}\, I \end{array}}$

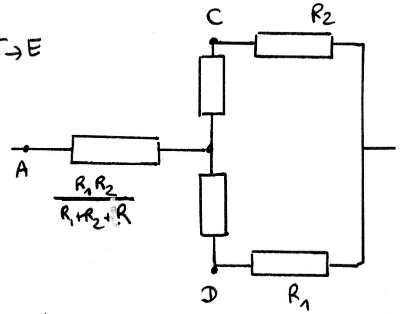
$$U_{AB} = R_{eq} I = U_{AC} + U_{CB} = R_1 I_1 + R_2 I_2 = \frac{R_1(R_2+R) + R_2(R_1+R)}{2R+R_1+R_2}\, I$$

$$\boxed{R_{\acute{e}q} = \frac{2R_1R_2 + RR_1 + RR_2}{2R + R_1 + R_2}}$$

b) Regroupement de résistances :

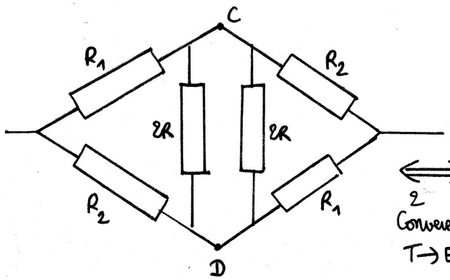


Conversion T→E $\Leftrightarrow$

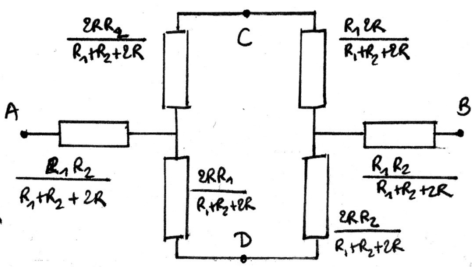


$\Updownarrow$

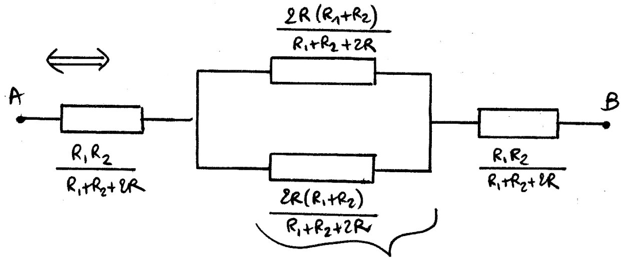
... Mais on brise la symétrie $\hookrightarrow$ Calculs lourds ...

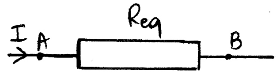


$\Longleftrightarrow$ 2 Conversion T→E



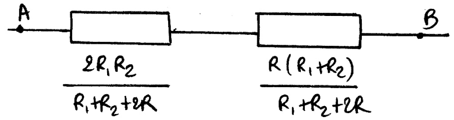
$\Longleftrightarrow$





$$\boxed{R_{eq} = \frac{2R_1R_2 + RR_1 + RR_2}{2R + R_1 + R_2}}$$

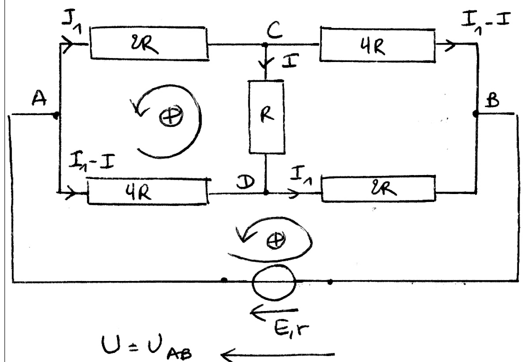
$\Longleftrightarrow$



2°) $U_{AB} = 11\,V = R_{eq} I_{A\to B} \rightarrow \boxed{I_{A\to B} = \dfrac{2R+R_1+R_2}{2R_1R_2 + RR_1 + RR_2}\, U_{AB}}$

$R_1 = 2R \quad R_2 = 4R \quad \boxed{I_{A\to B} = \dfrac{2R+2R+4R}{16R^2 + 2R^2 + 4R^2}\, U_{AB} = \dfrac{8}{22}\, U_{AB} = \dfrac{4}{11R}\, U_{AB}}$

AN $\boxed{I_{A\to B} = 4\ A}$ ... mais ce n'est pas la question demandée : on cherche $I = I_{C\to D}$.



Maille (A,D,C) : $-4R(I_1 - I) + RI + 2RI_1 = 0 \quad (1)$

Maille (A, géné, B,D,A) : $-U + 2RI_1 + 4R(I_1 - I) = 0 \quad (2)$

$\left\{\begin{array}{l} (1)\quad 5I - 2I_1 = 0 \\ (2)\quad -4RI + 6RI_1 = U \end{array}\right.$

(1) → $I_1 = \dfrac{5}{2} I$

(2) → $-4RI + 6R\dfrac{5I}{2} = U$

Soit $11RI = U \quad \boxed{I = \dfrac{U}{11R}}$ AN $\boxed{I = 1\ A.}$