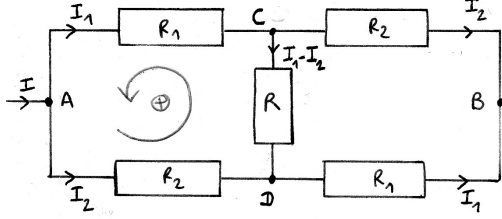


Ex-E2.9 : Résistance équivalente (3)



1) a) Lois de Kirchhoff :
 loi des noeuds en A (ou B) : $I = I_1 + I_2$ (1)
 En symétrie : $I_{A \rightarrow D} = I_2 = I_{C \rightarrow B}$
 $I_{A \rightarrow C} = I_1 = I_{D \rightarrow B}$
 loi des noeuds en D (ou C) : $I_{C \rightarrow D} = I_1 - I_2$

loi des mailles pour (A,D,C) : $-R_2 I_2 + R(I_1 - I_2) + R_1 I_1 = 0$

Système $\begin{cases} (1) I_1 + I_2 = I \\ (2) (R_1+R)I_1 - (R+R_2)I_2 = 0 \end{cases}$

→ par substitution :

$\begin{cases} (1) I_1 = I - I_2 \\ (2) (R_1+R)(I - I_2) - (R+R_2)I_2 = 0 \end{cases}$

$\hookrightarrow (R_1+R)I_1 - (R+R_2)I_2 = 0$ (2)

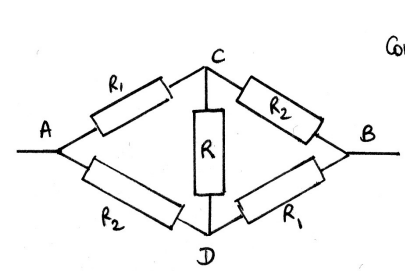
$$I_2 = \frac{R_1+R}{2R+R_1+R_2} I$$

$$I_1 = \frac{R_2+R}{2R+R_1+R_2} I$$

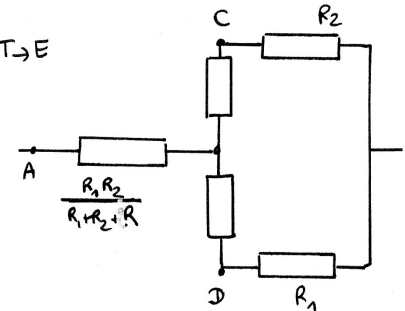
$$U_{AB} = R_{eq} I = U_{AC} + U_{CB} = R_1 I_1 + R_2 I_2 = \frac{R_1(R_2+R) + R_2(R_1+R)}{2R+R_1+R_2} I$$

$$R_{eq} = \frac{2R_1 R_2 + R R_1 + R R_2}{2R + R_1 + R_2}$$

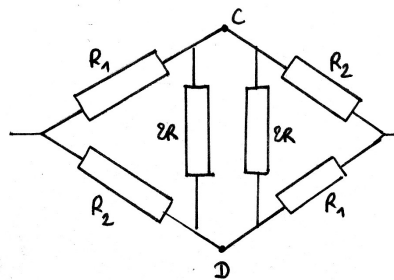
b) Regroupement de résistances :



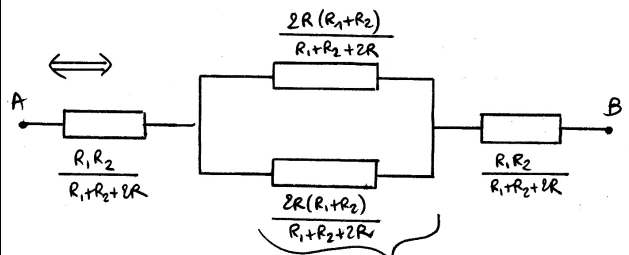
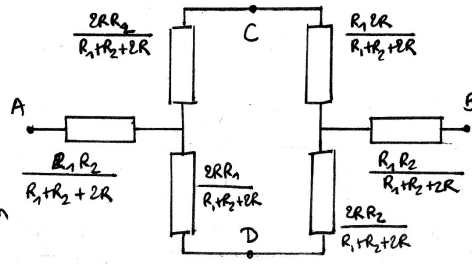
Conversion T → E



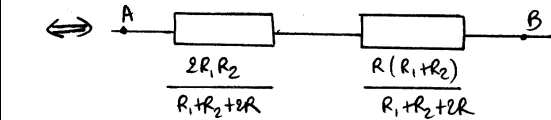
... Mais on brise la symétrie → Calculs lourds ...



Conversion T → E



$$R_{eq} = \frac{2R_1 R_2 + R R_1 + R R_2}{2R + R_1 + R_2}$$



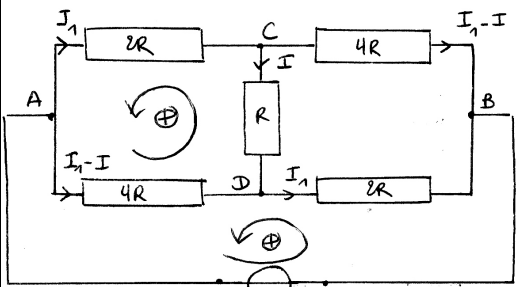
2) $U_{AB} = M V = R_{eq} I_{A \rightarrow B} \rightarrow I_{A \rightarrow B} = \frac{2R+R_1+R_2}{2R_1 R_2 + R R_1 + R R_2} U_{AB}$

$R_1 = 2R \quad R_2 = 4R$

$$I_{A \rightarrow B} = \frac{2R + 2R + 4R}{16R^2 + 2R^2 + 4R^2} U_{AB} = \frac{8}{22} U_{AB} = \frac{4}{11R} U_{AB}$$

AN $I_{A \rightarrow B} = 4 A$

... mais ce n'est pas la question demandée : on cherche $I = I_{C \rightarrow D}$.



Maille (A,D,C) : $-4R(I_1 - I) + RI + 2RI_1 = 0$ (1)
 Maille (A,gené,B,D,A) : $-U + 2RI_1 + 4R(I_1 - I) = 0$ (2)

$\begin{cases} (1) 5I - 2I_1 = 0 \\ (2) -4RI + 6RI_1 = U \end{cases}$

(1) → $I_1 = \frac{5}{2} I$

(2) → $-4RI + 6R \frac{5I}{2} = U$

Soit $MRI = U \quad I = \frac{U}{11R}$

AN $I = 1 A.$